

Models & Probabilities

Once upon a time, I drove out of the garage where my wife was working on something located very near our car. As I accelerated up to 55 mph on the highway, I suddenly saw something fly out in front of me near the windshield of the car. I watched in the rearview mirror as it fluttered past my driver side window and slid across the oncoming lane stopping where the white strip indicates the road shoulder.

In about 1 second I realized that the object skidding across the highway was my wife's mobile phone. And although the stretch of road I was driving on is heavily travelled, it was just lucky no oncoming car hit the phone while airborne. I drove for about one mile and turned around where it was safe thinking that, although the phone was most certainly destroyed, I could use its SIM card for a new one; this was probably not necessary but I was not sure.

When I got back to the phone as it lay on the road, it had not been driven over and smashed into a thin crepe. Besides not being smashed, I could see through a thoroughly cracked screen that it was still working; on top of that, my wife had put a special protective glass film on top of the face and that was what had cracked, the glass face was in perfect condition. It also has a thin plastic protective case around the back of the phone, but the case was undamaged. What was the probability that the phone would be undamaged save its protective covers, still usable? We know this to be a silly question to ask *ex post*, but suppose we asked it *ex ante*; what would we say about the probability it would not be destroyed if it was left on the hood of the car and driven out to the highway?

Could the probability be known *quantitatively* ahead of time beyond a guess? But if a guess were to be made (*ex ante*), it would most likely go heavily in favor of the phone's destruction, that would be my guess; there are so many ways disaster could have befallen it. It could be ruined just from the fall, the drop from the Honda Odyssey hood is fairly high and it landed on a very hard surface. Even it could be said it would survive the fall with fairly high likelihood, the conditions had to be just right for it to get out of harm's way; no car hit it as it fell, no car drove over it while it skidded on the road surface, and no car drove over it as it lay on the shoulder; it was not even in the actual shoulder because the white stripe is just where the shoulder starts.

Probably the thin glass protective cover helped by cracking and absorbing some of the energy of the fall; and it helped protect the screen from being scratched up. So maybe that would be taken into account, but the rest of the scenario is very hard to even think about assigning probabilities. Making correct probability "guesses" at failures in moderately complex settings, regardless of the method or "model" used to arrive at them, must be validated by repeating the actual event many times over and counting the destruction and non-destruction cases. That is, use the *law of large numbers* to create a "model."

Discussion For me it is interesting to think why *everyone knows how dice and coins work*; thinking about it, I imagine myself as a child who first encounters these objects and starts playing with them. A child, having no previous experience with such games, would have no reason to think any particular side would come up with some probability known beforehand. A precocious child may observe that, although after a few trials it is very difficult to know ahead of time which side will come up in a toss, she may begin to note how many times a toss produces particular results; she may observe the counts for each die face or each coin face are about the same.

In the middle of the 16th century, Gerolamo Cardano wrote the book "Liber de Ludo Aleae" whose title is roughly translated to English as Liber: "Book," de: "About," Ludo: "Games" Aleae: "Chance"; or Book about Games of Chance.¹ My understanding is Cardano was the first mathematician who thought about investigating how probability works in light of knowing the outcome of many trials "on average." Late in the 17th century, Jacob Bernoulli formally stated the principle that indicates if many coin tosses are made properly, that is they bounce and flip around randomly when tossed, the closer the number of heads and number of tails will get to being equal. This became the principle behind "The Law of Large Numbers"; it is because of this principle we know that the chance of heads coming up on a fair coin toss is 50%.² More importantly, it has some serious implications for investigators looking into failure probabilities for moderately complex systems for which little or no failure data are available.

¹Cardano, G. (c. 1564). Liber de Ludo Aleae. Opera Omnia, Vol. 1, 1663: translated to English for example in Cardano, G., 1961. The book on games of chance: (Liber de ludo aleae). Holt, Rinehart and Winston.

²Well mostly, see Diaconis, P., Holmes, S. and Montgomery, R., 2007. Dynamical bias in the coin toss. SIAM review, 49(2), pp.211-235.